Karachi university business school
University of Karachi
Fiscal Examination, June 2011: Affiliated Colleges
Business Mathematics-BA (H)-322
BB-II

Date: June 11, 2010
Max Marks: 80
Max Time: 3hrs

Instructions: Attempt all questions in sequence preferably.

Q.1. A dietitian is planning a menu for the evening meal at university dining hall. Three main items will be served, all having different nutritional contents. The dietitian is interested in providing at least the minimum daily requirements (MDR) of each of three vitamins in the one meal. The table below summarizes the vitamin content per ounce of each type of food, the cost per ounce of each food and MDR for the three vitamins. Any combination of foods may be served as long as the total serving size is at least 5 ounces. The problem is to determine the number of ounces of each food to be included in the meal. The objective is to minimize the cost of each meal subject to satisfying the MDR requirements of the three vitamins as well as the restriction on minimum serving size. Formulate the linear programming model for this problem. (Note: Only give complete formulation).

<table>
<thead>
<tr>
<th>Food</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Cost per ounce ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>20</td>
<td>10</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>10</td>
<td>50</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>30</td>
<td>20</td>
<td>0.12</td>
</tr>
<tr>
<td>MDR</td>
<td>290</td>
<td>200</td>
<td>340</td>
<td></td>
</tr>
</tbody>
</table>

Q.2. Transform the following set into the standard form and solve by the simplex method.

\[ \begin{align*}
    2x_1 + 3x_2 & \leq 100 \\
    2x_1 + 3x_2 & \leq 40 \quad (i) \\
    x_1, x_2 & \geq 0
\end{align*} \]

Q.3. Evaluate the following:
(i) \( \lim_{x \to \infty} \left\{ x \left( x^2 + 10 \right) \right\} \)
(ii) \( \lim_{x \to 3} (3x^2 - 4x + 10) \)
(iii) Determine the values which make the given function discontinuous: \( f(x) = \frac{1}{x^2 - 1} \)
(iv) Determine the general expression for the difference quotient of the function \( f(x) = x^2 \)
(v) Find all higher order derivatives for \( f(x) = 25x^6 + 9x^4 - 10x + 4 \)

Q.4. Examine the following function for any critical points and determine their nature. \( f(x) = e^{-x^2} \)

Q.5. Determine the following:
(i) \( \int x^e \, dx \)
(ii) \( \int (6x^3) \, dx \)
(iii) \( \int 67 \, dx \)
(iv) The marginal revenue function for a company's product is \( MR = 40000 - 4x \) (\( x \) = number of units sold). Find the total revenue function for the product.
(v) \( \int f(x) \, dx \)

End of Paper
KARACHI UNIVERSITY BUSINESS SCHOOL
(University of Karachi)
FINAL EXAMINATION DECEMBER 2010: AFFILIATED COLLEGES
BUSINESS MATHEMATICS – H BA (H) – 322
BS – 11

Instructions:
1. Attempt all questions in sequence. All questions carry equal marks.
2. Exchange of stationary items/calculators & use of Mobile Phone is strictly prohibited.

Date: Dec 22, 2010 Time: 3 Hours Max. Marks: 60

Q.1. Attempt the following:
   (i) Differentiate Continuous function and discontinuous function
   (ii) Differentiate Tangent line and secant line
   (iii) The slope of a pure horizontal line is
   (iv) Discuss the application of linear programming from a business perspective
   (v) Differentiate between Transportation model and Assignment model

Q.2. Solve the following linear programming problem by the corner point method.

Maximize \( z = 20x_1 + 15x_2 \)
Subject to
\[ \begin{align*}
3x_1 + 4x_2 & \leq 60 \\
4x_1 + 3x_2 & \leq 60 \\
x_1 & \leq 10 \\
x_1, x_2 & \geq 0
\end{align*} \]

Q.3. Determine the solution and values of the absolute maximum and absolute minimum for the following function and also sketch the function: \( f(x) = 2x^2 - 4x + 5 \) where \( 2 \leq x \leq 8 \)

Q.4(a) Integrate the following:
   i. \( \int x^2 \, dx \)
   ii. \( \int (x^2 - 2x) \, dx \)
   iii. \( \int 2x^3 \, dx \)
   iv. \( \int (3x^2 - 10) \, dx \)
   v. \( \int \ln x \, dx \)

Q.4(b) Determine discontinuities (if any):
   (i) \( f(x) = 4x^3 \)
   (ii) \( f(x) = 4x^{3/2} \)
   (iii) \( f(x) = 2x^4 + 4x^{1/2} \)
   (iv) \( f(x) = 2x^2 + 4x + 1.5x + 12 \)

Q.5(a) Determine \( f(x) \) for the following:
   (i) \( f(x) = x^3 - 5x - x^{-3} \)
   (ii) \( f(x) = e^x \)
   (iii) \( f(x) = \ln(5x^2 + 2x + 1) \)
   (iv) \( f(x) = e^{x^2} \)
   (v) \( f(x) = (10 - x)(x^2 + 1) \)

Q.5(b) A ball thrown upwards from the roof of a building which is 600 feet high will be at a height of \( h \) feet after \( t \) seconds, as described by the function: \( h = f(t) = -16t^2 + 50t + 600 \)
   i. What is the height of the ball after 3 seconds
   ii. What is the velocity of the ball after 3 seconds
   iii. What is the acceleration of the ball at \( t = 07 \) s?
Q1. Define the following:

i. A function
ii. Domain of a function
iii. Maxima under the curve \( y = f(x) \) from a to b
iv. Slope of a line passes through two points
v. Critical point
vi. Continuous function
vii. Inflection points
viii. Differential equation
ix. Differentiable function
x. Infinite integral
xi. Limit of a function
xii. Feasible solution

Q2 (a) Using limit approach find \( f'(x) \) if \( f(x) = x^3 - 3x^2 \)

Q2 (b) The demand for the product of a firm varies with the price that the firm charges for the product. The firm estimates the annual total revenue \( R \) (stated is $1,000) is a function of the price \( p \) (stated in $).

\[
R = \frac{2p^3}{3} - \frac{15p^2}{2} + 20p + 100
\]

i. Determine the price should be charged in order to maximize total revenue.
ii. What is the maximum value of annual total revenue?

Q3 (a) Find \( f(2) \), \( f'(2) \) if \( f(x) = \frac{e^{3x}}{x^2 - 1} \)

Q3 (b) Solve the Differential Equation

\( f''(x) = x - 5, f'(2) = 4, f(0) = 10 \)

Q4 Integrate the following:

i. \( \int xe^x \, dx \)
ii. \( \int \tan^2 \theta \, d\theta \)
iii. \( \int \sqrt{x} + 1 \, dx \)

Q5 Graph the region of feasible solution of the following L.P. problem and solve by the corner point method

Maximize \( z = 4x + 8y \)

Subject to

1. \( 2x + y \leq 30 \)
2. \( x + 2y \leq 24 \)
3. \( x, y \geq 0 \)
KARACHI UNIVERSITY BUSINESS SCHOOL
University of Karachi

FINAL EXAMINATION, DECEMBER 2009; AFFILIATED COLLEGES
BUSINESS MATHEMATICS B.A. (P) -- 322
FIVE YEARS PROGRAM

Date: December 21, 2009
Max Marks: 60
Max Time: 3 Hours

Instructions:
1. Attempt all questions in sequence preferably but not necessary
2. All questions carry equal marks
3. Exchange of stationery items/calculators & use of Mobile Phone is strictly prohibited
4. Financial tables of any kind shall not be provided

Q. No.1
Distinguish between the following using supportive examples:
(a) Real Number and complex number
(b) Continuous function and discontinuous function
(c) Tangent line and secant line
(d) Differentiation and Integration
(e) Inflection point and critical point

Q. No.2(a)
(a) Find dy/dx if y = f(x) = 5u + 3 and v = g(x) = -3x + 10
(b) f(x) = x^2 - 2x + 5
(c) f(x) = ln(x)
(d) f(x) = (x^2 - 5)^3
(e) Find dy/dx = (x-1)/ln x

Q. No.2(b)
The population of a country is estimated by the function
P = 125 e^{0.03t}
where P equals the population (in million) & t equals time measured in years since 1990.
(i) What is the population expected to equal in the year 2000?
(ii) Determine the expression for the instantaneous rate of change in the population.
(iii) What is the instantaneous rate of change in the population expected to equal in the year 2001?

Q. No.3
Examine the function for any critical point and determine their nature also sketch the function.
f(x) = (x^2/4) - (9x^2/2)

Q. No.4(a)
The function describing the marginal profit from producing and selling a product is MP = 6x + 450, where x equals the number of units and MP is the marginal profit measured in dollars. When 100 units are produced and sold total profit equals $5,000. Determine the total profit function.

Q. No.4(b)
Integrate the following:
1. \( \int \frac{x^2}{2} \, dx \)
2. \( \int (3x-6) \, dx \)
3. \( \int \sqrt{x-30} \, dx \)
4. \( \int (x^2-2x+2) \, dx \)
Classify the following differential equation by order and degree.
(a) \( \frac{dy}{dx} = x^2 - 2x + 1 \)
(b) \( \frac{dy}{dx} + (2x)(dy/dx) = x^2 \)

Q. No.5
Solve the following linear programming problem by the corner point method.
Maximize \( z = 4x_1 + 3x_2 \)
Subject to
\( x_1 + x_2 \leq 20 \)
\( 2x_1 + x_2 \leq 32 \)
\( x_1, x_2 \geq 0 \)
Instructions:
1. Attempt all questions in sequence preferably but not necessary
2. All questions carry equal marks.
3. Exchange of stationary items/calculator & use of Mobile Phone is strictly prohibited.
4. Financial tables of any kind shall not be provided.

Dated: Jun 11, 2009  Time: 180 Minutes  Max. Marks: 60

Q.1. Solve the following linear problem by using the simplex method.

Maximize \[ z = 2x_1 + 12x_2 + 8x_3 \]
Subject to \[ 2x_1 + 2x_2 + x_3 \leq 100 \]
\[ x_1 + 2x_2 + 5x_3 \leq 300 \]
\[ 10x_1 + 5x_2 + 4x_3 \leq 300 \]
\[ x_1, x_2, x_3 \geq 0 \]

Q.2. (a) Use the graph of the function given below to determine the indicated limits.

(i) \( \lim_{x \to -5^-} f(x) \)  (ii) \( \lim_{x \to -5^+} f(x) \)  (iii) \( \lim_{x \to 5^-} f(x) \)  (iv) \( \lim_{x \to 5^+} f(x) \)  (v) \( \lim_{x \to 0^-} f(x) \)  (vi) \( \lim_{x \to 0^+} f(x) \)
Q.2(b) Find the indicated limits
(i) \( \lim_{x \to 175} (x^3 - x)(5x + 100) \)
(ii) \( \lim_{x \to 0} \frac{3x}{x^3 + 3x^2 - x} \)
(iii) \( \lim_{x \to \infty} \frac{1 + 3x}{x^2} \)

Q.3(a) A ball is thrown straight up into the air. The height of the ball can be described as a function of time according to the function
\[ h(t) = -16t^2 + 128t \]
Where \( h(t) \) is height measured in feet and \( t \) is time measured in seconds.
(i) Determine the average rate of change in height between \( t = 0 \) and \( t = 2 \). Between \( t = 0 \) and \( t = 4 \). Between \( t = 0 \) and \( t = 8 \).
(ii) How long does it take for the ball to hit the ground? (\( h = 0 \))?

Q.3(b) Determine all higher order derivatives.
(i) \( f(x) = e^{2x} \sin x \)
(ii) \( f(x) = (x^3 - 1)^3(6x - 5) \)
(iii) \( f(x) = \ln(5x) \)
(iv) \( f(x) = x^3 + 10e^x \)
(v) \( f(x) = (x^2 - 3x + 5)^8 \)

Q.4. Examine the following function for any critical point and determine their nature.

\[ f(x) = -x^2 \]

Q.5. The annual profit for a firm depends upon the number of units produced. Specifically, the function which describes the relationship between profit \( P \) (in Indian Rupees) and the number of units produced \( x \) is:
\[ P = 0.01x^2 - 500x + 25000 \]
(i) Determine the number of units \( x \) which will result in maximum profit
(ii) What is the expected maximum profit?

Q.6(a) A particular prescription drug was administered to a person in a dosage of 10mg.
The amount of the drug content in the bloodstream diminishes over time, as described by an exponential decay function. After 6 hours, a blood sample reveals that the amount in the system is 40mg. If \( V \) equals the amount of the drug in the bloodstream after \( t \) hours and \( V \) equals the amount in the bloodstream at \( t = 0 \), then state the particular function describing the drug level decay function.

Q.6(b) Find the general and particular solution for the following differential equations.
(i) \( \frac{dy}{dx} = 3x, f(0) = -20 \)
(ii) \( a_2y'' + 6x - 9, f(2) = 10, f(-2) = -10 \)
KARACHI UNIVERSITY BUSINESS SCHOOL
UNIVERSITY OF KARACHI
FINAL EXAMINATION, DECEMBER 2008: AFFILIATED COLLEGES
BUSINESS MATHEMATICS – II: BA (H) – 322
BS - II

Date: December 23, 2008
Time allowed: 3 Hours
Max Marks: 60

Instructions:
1. Attempt all questions in sequence preferably but not necessary
2. All questions carry equal marks.
3. Exchange of stationary items/calculators & use of Mobile Phones is strictly prohibited.
4. Financial tables of any kind shall not be provided

Dated: Dec 23, 2008
Time: 150 Minutes
Max. Marks: 60

Q.1. Solve the following:
   a) (x-3)-1 = (3x-9)/3
   b) x^2 - 2 = 7
   c) Sketch the function: 2x + 3y = 0
   d) Identify the following as continuous or discontinuous function:
      i) f(x) = 3x^2 + 4x - 3x + 2 = 0
      ii) f(x) = x(x+4)
      (c) Identify the following graphical functions as continuous or discontinuous:

Q.2(a) Determine the limit of the function: \[ \lim_{x \to 4} \sqrt[3]{x^2 - 5x + 1} \]
   b) Define the following:
      i) Tangent line
      ii) Secant line

Q.2(b) Determine \( f(x) \) for the following:
   i) \( f(0) = x^2 \)
   ii) \( f(x) = e^x \)
   iii) \( f(x) = \ln(x^2 - 3x + 3) \)
   iv) \( f(x) = x^3 \)
   v) \( f(x) = e^{-x} \)

Q.2(c) Find all higher-order derivatives:
   i) \( f(x) = x^3 - 2x^3 \)
   ii) \( f(x) = e^x \)
   iii) \( f(x) = 200514.231 \)

Q.2(d) For the following function use \( f''(x) \) to determine the concavity conditions at \( x = 2 \) and \( x = 1 \)
   (i) \( f(x) = x^4 + 3x^3 + 2 \)
   (ii) \( f(x) = \sqrt{x - 10} \)
   (iii) \( f(x) = e^x \)

Q.3(a) For the following function identify the location of any inflection point:
   i) \( f(x) = (x-5)^3 \)
   (ii) \( f(x) = x^3 \)
   (iii) \( f(x) = 10x^4 + 100 \)
Q.4 Examine the following function for any critical points and determine their nature.
\[ f(x) = 2x^2 - 12x - 10 \]
Where \( 25 < x < 10 \) also sketch the function.

Q.5 A retailer of motorbike has examine cost data and determine a cost function which expresses
the annual cost of purchasing, owning and maintaining inventory as function of the size (in.
of the unit) of each order \( q \) places for the motorbike. The cost function is

\[ C = f(q) = 4800 + 15q + 750000 \]

where \( C \) = annual inventory cost and \( q \) = no. of bicycle order each time.

Requirement:
(a) Determine the order size which minimize the annual inventory cost
(b) What is minimum annual inventory cost expected to equal

Q.6(a) Integrate the following

\[ \int \left( \frac{2x^3}{3x^2} + 2 \right) dx \]

\[ \int x^3 - 3x^2 + (x-1) dx \]

\[ \int -5x \ dx \]

\[ \int (x^3 - 2x^2 + 4) dx \]

\[ \int (x-1)^3 (x^2 + x + 10) dx \]

Q.6(b) An LP problem has 15 decision variables, 20 (\( < \)) constraints, 12 (\( > \)) constraints and 8 (\( = \)) constraints. When rewritten in standard form, how many variables will be included? How many supplemental variables of each type?
Q.1(a) Graphically determine the permissible half space which satisfies the inequality

(i) \(-x + 2y \geq 8\)

(ii) \(-2x + 4y \leq 24\)

(b) Graphically determine the solution space (if exists)

(i) \(2x - 4y \leq 20\)

(ii) \(3x + 2y \leq 32\)

(c) Describe the following (Illustrate graphically where necessary)

(i) Optimal solution

(ii) Alternative optimal solution

(iii) Unbounded solution

(d) Refer to the diagram at the end of this paper and answer the questions.

Q.2(a) Solve the following linear programming problem by the corner point method.

Maximize \[ z = 20x_1 + 15x_2 \]

Subject to

\[ 3x_1 + 4x_2 \leq 60 \]

\[ 4x_1 + 3x_2 \leq 60 \]

\[ x_1 \leq 10 \]

\[ x_2 \leq 12 \]

\[ x_1, x_2 \geq 0 \]

Q.2(b) Given the above L.P problem rewrite the constraint set into standard form incorporating all supplemental variables.

Q.3(a) A person recently won a lottery. The terms of the lottery are that the winner will receive annual payments of Rs20,000 at the end of this year and each of the following 3 yrs. If the winner could invest money today at a rate of 8 percent per year compounded annually what is the present value of the four payments.

Q.3(b) Determine whether the investment project depicted by the cash flow diagram satisfies the minimum desired rate of return criterion. What is the NPV of the indicated interest rate? (All amount are in Pak Rupees). Minimum rate of return = 10% per yr.

\[
\begin{align*}
200,000 & \quad \text{time in yrs} \\
1 & \quad 2 \quad 3 \quad 4 \quad 5 \\
1,000,000 & 
\end{align*}
\]
Q.4(a) Integrate the following:

i. \( x^3 \) \( dx \)

ii. \( 3x^2 - 2x \) \( dx \)

iii. \( 2x + 5 \) \( dx \)

iv. \( \frac{1}{3x^2 - 10} \) \( dx \)

v. \( \int \frac{dx}{x^2 - 2x + 1} \)

vi. \( \int \frac{dx}{x} \)

Q.4(b) Determine all discontinuities (if any)

(i) \( f(x) = 3x^2 \)

(ii) \( f(x) = \frac{x^2}{4} \)

(iii) \( f(x) = 30.5 \)

(iv) \( f(x) = 2x^4 + 4x^2 + 1.5x + 12y \)

Q.5. A retailer of motorbikes has examined past data and determines a cost function which expresses the annual cost of purchasing, owning and maintaining inventory as function of the size \( q \) of the order of each order \( q \) places for the motorbike. The cost function is:

\[ C = f(q) = 4860/q + 15q + 750000 \]

where \( C \) = annual inventory cost and \( q \) = no. of motorbike ordered each time.

Requirement:

(a) Determine the order size which minimize the annual inventory cost

(b) What is minimum annual inventory cost expected to equal?

Q.6. Determine the area bounded by \( f(x) = 5x + 7 \) between \( x = 3 \) and \( x = -2 \).

Q.7. For the function given below: \( f(x) = 3x^3 - 4x + 100 \)

(a) Determine the location of all critical points and determine their nature.

(b) Plot the graph of the function.

(c) Identify the location and values of the absolute maximum and absolute minimum for the following function.

\[ f(x) = 2x^2 - 4x + 5 \]

where \( 2 \leq x \leq 2 \)

Q.8.1(d)

For the given region of feasible solution:

(a) Identify the nature of each constraint.

(b) How many basic & non-basic variables will be there in any basic feasible solution?

(c) What are the basic & non-basic variables associated with corner points A, B, C & D?
KARACHI UNIVERSITY BUSINESS SCHOOL
UNIVERSITY OF KARACHI
Final Examination: Affiliated Colleges
Business Mathematics – II BA(P) – 342

Dated: June 72, 2007
Time: 3hrs.
Max. Marks: 60

Instructions: Attempt all questions, all question carry equal marks.
Attempt question in sequence.

Q.1. Maximize
\[ Z = 2x_1 + 12x_2 + 8x_3 \]
Subject to
\[ 2x_1 + 12x_2 + x_3 \leq 100 \]
\[ x_1 + 2x_2 + 5x_3 \leq 80 \]
\[ 3x_1 + 5x_2 + 4x_3 \leq 100 \]
\[ x_1, x_2, x_3 \geq 0 \]

Q.2. A beverage company has four plants in Karachi, Lahore, Peshawar, and Quetta, and it must ship its finished product to its warehouses in Rawalpindi, Multan, Hyderabad, Faisalabad and Sukkur, the units shipping cost, availability at the factories, and requirements at the warehouses are shown in the table:

<table>
<thead>
<tr>
<th>Plant</th>
<th>Multan</th>
<th>Hyd.</th>
<th>Faisal.</th>
<th>Sukkur</th>
<th>Rawalpindi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Karachi</td>
<td>10</td>
<td>50</td>
<td>22</td>
<td>8</td>
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<tr>
<td>Lahore</td>
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<td>32</td>
<td>25</td>
<td>22</td>
<td>25</td>
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<tr>
<td>Quetta</td>
<td>8</td>
<td>15</td>
<td>24</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>Faisal.</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>40</td>
</tr>
</tbody>
</table>

Find out the feasible solution (Transportation Proc.)

Q.3. A new State welfare agency wants to determine how many analysts to hire for processing of welfare applications. It is estimated that the average cost C of processing an application is a function of the number of analysts : N

\[ C = f(N) = \frac{N^2}{3} + \frac{2N}{3} + 6N + 29 \]

i) If the objective is to minimize the average cost per application, determine the number of analysts who should be hired.
ii) What is the minimum average cost of processing an application expected.
Q.3.(b) What will be the present value of a continuous income stream of Rs. 3500 per annum for four years, if it is discounted continuously at the rate of 0.05 per year.

Q.4.(b) The market demand law of a firm is given by \( 4p + q - 16 = 0 \), find the output level when the revenue is maximum and find that maximum revenue.

(b) If \( Z = \frac{x}{\sqrt{x^2 + y}} \), find partial derivatives of \( Z \) w.r.t. \( x \) and \( y \) respectively.

Q.5. Integrate

(i) \( \int e^x \cos x \, dx \)

(ii) \( \int \frac{3x^4 - 5x}{(2x^2 - 5x^2 + 15)^2} \, dx \)

Q.6.(a) Determine the area bounded by the curve \( y = \frac{1}{\sqrt{8-x}} \) and x-axis and \( x = 8 \).

(b) Given the demand function, \( q_1 = f(p_1, p_2) = 250,000 - 0.5p_1^2 + p_2 \), \( p_1 \), and \( p_2 \).

i) Determine the partial derivatives \( f_1(p_1, p_2) \) and \( f_2(p_1, p_2) \).

ii) If \( p_1 = 30 \), \( p_2 = 10 \) and \( p_3 = 20 \), evaluate the partial derivatives and interpret their meaning.
KARACHI UNIVERSITY BUSINESS SCHOOL
BBA 2nd Semester - 2005-06

Time: 3 Hours
Business Mathematics II (C # 352)
Max. Marks: 60

ATTEMPT ALL QUESTIONS, CARRY EQUAL MARKS.

Q. 1. Minimize
   \[ Z = 0.10x_1 + 0.15x_2 + 0.12x_3 \]
   Subject to
   \[ 50x_1 + 30x_2 + 20x_3 \geq 299 \]
   \[ 20x_1 + 30x_2 + 30x_3 \geq 200 \]
   \[ 10x_1 + 50x_2 + 29x_3 \geq 216 \]
   \[ x_1 + x_2 + x_3 \geq 9 \]
   \[ x_1 + x_2 + x_3 \geq 0 \]

Q. 2. A beverage company has four plants in Karachi, Lahore, Peshawar and Quetta, and it must ship its finished product to its warehouses in Rawalpindi, Multan, Hyderabad, Faisalabad and Sukkur, with the unit shipping cost, availabilities at the factories, and requirements at the warehouses shown in table.

<table>
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<tr>
<th></th>
<th>Multan</th>
<th>Hydra</th>
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<th>Sukkur</th>
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<tr>
<td>R. Pindi</td>
<td>2</td>
<td>17</td>
<td>10</td>
<td>24</td>
<td>7</td>
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<tr>
<td>Warehouse requirement</td>
<td>10</td>
<td>50</td>
<td>20</td>
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</tbody>
</table>

Find out the feasible solution (Transportation Prob.)

Q. 3(a) A new state welfare agency wants to determine how many analysts to hire for processing of welfare applications. It is estimated that the average cost \( C \) of processing an application is a function of the number of analysts \( x \).

Specifically, the cost function is \( C = f(x) = \frac{x^3}{3} - \frac{7x^2}{2} + 6x + 29 \)

(i) If the objective is to minimize the average cost per application, determine the number of analysts who should be hired.

(ii) What is the minimum average cost of processing an application expected?

(b) A ball is thrown straight up into the air. The height of the ball can be described as a function of time according to function \( h(t) = -16t^2 + 128t \), where \( h(t) \) is height measured in feet and \( t \) is time measured in seconds.

(i) What is the height 2 seconds after the ball is thrown?
(ii) When will the ball attain its greatest height?
(iii) When will the ball hit the ground?

Could .....P2 .....
**Q4(a) (Minimize average cost per unit) the total cost of producing unit of a certain product is described by the function.**

\[ C = 180,000 + 1,500q + 0.2q^2 \]

Where \( C \) is the total cost stated in dollars. Determine the number of units of \( q \) that should be produced in order to minimize the average cost per unit.

**Q4(b) The function describing the marginal profit from producing and selling a product is \( MP = -3x + 500 \).**

Where \( x \) equals the number of units and \( MP \) is the marginal profit measured in dollars when 200 units are produced and sold, total profit equals $15,000. Determine the total profit function.

**Q5(a) Integrate**

(i) \[ \int \frac{(x+2)dx}{(2x^2+3x)} \]

(ii) \[ \int x^2 \ln x \, dx \]

**Q5(b) Determine the size of the area bounded by function, the X-axis over the indicated interval of \( f(x) = 4xe^{-x^2} \), between \( x = 1 \), and \( x = 3 \).**
Q.1(a) Graphically determine the permissible half-space which satisfies the inequality.
   (i) \(-x + 2y \geq 8\)  
   (ii) \(-2x + 6y \leq 24\)

(b) Graphically determine the solution space (if exists)
   (i) \(2x - 4y \leq 20\)  
   (ii) \(5x + 2y \leq 20\)  
   \(3x + 4y \leq 32\)

(c) Describe the following (Illustrate graphically where necessary)
   (i) Optimal solution
   (ii) Alternative optimal solution
   (iii) Unbounded solution
   (d) Refer to the diagram at the end of this paper and answer the question.

Q.2(a) Solve the following linear programming problem by the corner point method.

Maximize \(z = 20x_1 + 15x_2\)
Subject to
\[
\begin{align*}
3x_1 + 4x_2 &\leq 60 \\
4x_1 + 3x_2 &\leq 60 \\
x_1 &\leq 10 \\
x_1, x_2 &\geq 0
\end{align*}
\]

Q.2(b) Given the above LP problem rewrite the constraint set into standard form incorporating all supplemental variables.

Q.3(a) A person recently won a lottery. The terms of the lottery are that the winner will receive annual payments of Rs20,000 at the end of this year and each of the following 5 yrs. If the winner could invest money today at the rate of 8 percent per year compounded annually what is the present value of the four payments.

Q.3(b) Determine whether the investment project depicted by the cash flow diagram satisfies the minimum desired rate of return criteria. What is the NPV at the indicated interest rate? (All amount are in Pak Rupees). Minimum rate of return=10% per yr

![Cash Flow Diagram]

\[
\begin{array}{cccccc}
200,000 & -200,000 & 1,000,000 \\
1 & 2 & 3 & 4 & 5 & \text{time in yrs}
\end{array}
\]
Q.4(a) Integrate the following:

i. \( \int x^3 \, dx \)

ii. \( \int x^2 - 2x^3 \, dx \)

iii. \( \int 2x^4 \, dx \)

iv. \( \int x \, (x^2 - 10) \, dx \)

v. \( \int \sqrt{x} \, dx \)

vi. \( \int (5x^2 - 2x + 3) \, dx \)

vii. \( \int \frac{1}{x} \, dx \)

Q.4(b) Determine discontinuities (if any)

(i) \( f(x) = 3x^3 \)

(ii) \( f(x) = \frac{x}{x^2 - 1} \)

(iii) \( f(x) = 3x + 2 \)

(iv) \( f(x) = \frac{1}{x} \)

(v) \( f(x) = g(x) \times h(x) \)

Q.5. A retailer of motorbikes has examined cost data and determines a cost function which expresses the annual cost of purchasing, owning and maintaining inventory as function of the size (no. of the units) of each order it places for the motorbike. The cost function is:

\[ C = f(q) = 4800q + 15q^2 + 75000 \]

where \( C \) = annual inventory cost and \( q \) = no. of motorbikes ordered each time.

Requirement:

(a) Determine the order size which minimizes the annual inventory cost

(b) What is minimum annual inventory cost expected to equal?

Q.7. Determine the area, indicated by \( f(x) = 5x + 7 \) between \( x = 3 \) and \( x = -2 \).

Q.7. For the function given below:

\[ f(x) = 3x^2 - 48x + 100 \]

(a) Determine the location of all critical points and determine their nature.

(b) Sketch the graph of the function.

(c) Determine the location and values of the absolute maximum and absolute minimum for the following function.

\[ f(x) = 2x^3 - 4x + 5 \]

where \( x \geq 3 \)

Q.1 (d)

For the given region of feasible solution:

(a) Identify the nature of each constraint.

(b) How many basic & non-basic variables will be there in any basic feasible solution?

(c) What are the basic & non-basic variables associated with corner points A, B, C & D?
Instructions:
1. Attempt all questions in sequence. All questions carry equal marks.
2. Exchange of stationery items/registration & use of Mobile Phone is strictly prohibited.

Dated: Dec 20, 2005   Time: 180 Minutes   Max. Marks: 60

Q.1(a) Graphically determine the permissible half-space which satisfies the inequality

(i) -x + 2y > 8
(ii) -2x + 6y ≤ -24

Q.1(b) Graphically determine the solution space (if exists)

(i) 2x - 4y ≤ 20
   3x + 2y ≤ 32

Q.1(c) Describe the following (trust the graphically where necessary)

(i) Optimal solution
(ii) Alternative optimal solution
(iii) Unbounded solution

Q.2. Solve the following linear programming problem by the corner point method.

Maximize \( z = 30x_1 + 15x_2 \)

Subject to

\[ 3x_1 + 4x_2 \leq 60 \]
\[ 4x_1 + 3x_2 \leq 60 \]
\[ x_1 \leq 10 \]
\[ x_2 \leq 12 \]
\[ x_1, x_2 \geq 0 \]

Q.3(a) A person recently won a lottery. The terms of the lottery are that the winner will receive annual payments of Rs250,000 at the end of this year and each of the following 3 yrs. If the winner could invest money today at the rate of 8 percent per year compounded annually what is the present value of the four payments?

Q.3(b) Determine whether the investment project depicted by the cash flow diagram satisfies the minimum desired rate of return criterion. What is the NPV at the indicated interest rate? (All amount are in Pak Rupees). Minimum rate of return=10% per yr

\[ \text{Time in yrs} \]

\[ \text{Cash flows} \]

\[ 200,000 \]

\[ 1 \quad 2 \quad 3 \quad 4 \quad 5 \]

\[ 1,000,000 \]
Q.4(a) Integrate the following:

i. \( x^4 \, dx \)

ii. \((x^3 - 2x)^5 \, (x-1) \, dx \)

iii. \(2x^{13} \, dx \)

iv. \(6x / (3x^2-10) \, dx \)

Q.4(b) Determine discontinuities (if any)

(i) \( f(x) = 3x^3 \)
(ii) \( f(x) = x^4 / 4 \)
(iii) \( f(x) = 39.5 \)
(iv) \( f(x,y) = 2x^4 + 4x^2 + 1.5x + 12y \)

Q.5. A retailer of motorbike has examined cost data and determines a cost function which expresses the annual cost of purchasing, owning and maintaining inventory as function of the size (no. of the unit) of each order it places for the motorbike. The cost function is

\[ C = f(q) = 4800/q + 15q + 750000 \]

where \( C = \) annual inventory cost and \( q = \) no. of motorbike ordered each time.

Requirement:

(a) Determine the order size which minimize the annual inventory cost
(b) What is minimum annual inventory cost expected to equal?

Q.6. Determine the area indicated by \( f(x) = x + 5 \) between \( x = 3 \) and \( x = -2 \). (by both the method)

Q.7. For the function given below: \( f(x) = 3x^2 - 4x + 100 \)

a) Determine the location of all critical points and determine their nature
b) Sketch the graph of the function
c) Determine the location and values of the absolute maximum and absolute minimum for the following function.

\[ f(x) = 2x^2 - 4x + 5 \]

where \( 2 \leq x \leq 8 \)
KARACHI UNIVERSITY BUSINESS SCHOOL
UNIVERSITY OF KARACHI
Final Examination, Affiliated Colleges
Business Mathematics – II BA(P) – 342

Date: June 12, 2007
Max. Marks: 60
Time: 3 hrs.

Instructions: Attempt all questions. All questions carry equal marks.
Attempt question in sequence.

Q.1. Maximize
\[ Z = 2x_1 + 12x_2 + 8x_3 \]
\[ 2x_1 + 12x_2 + 3x_3 \leq 100 \]
\[ x_1 + 4x_2 + 5x_3 \leq 80 \]
Subject to
\[ 10x_1 + 5x_2 + 4x_3 \leq 300 \]
\[ x_1, x_2, x_3 \geq 0 \]

Q.2. A beverage company has four plants in Karachi, Lahore, Peshawar and Quetta, and it must ship its finished product to its warehouses in Peshawar, Multan, Hyderabad and Sukkur. The table shipping costs, availabilities at the factories, and requirements at the warehouses are shown in the table.

<table>
<thead>
<tr>
<th></th>
<th>Multan</th>
<th>Hyderabad</th>
<th>Sukkur</th>
<th>WP</th>
<th>Factories Available</th>
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<tbody>
<tr>
<td>Karachi</td>
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<td>3</td>
<td>13</td>
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<tr>
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<td>10</td>
<td>50</td>
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<td>80</td>
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</tr>
</tbody>
</table>

Find out the feasible solution (Transportation Prob.).

Q.3. (a) A new state welfare agency wants to determine how many analysts to hire for processing of welfare applications. It is estimated that the average cost \( A \) of processing an application is a function of the number of analysts \( x \).

Special case the cost function is \( A = f(x) = \frac{x^2}{3} + \frac{7x^2}{2} + 6x + 29 \)

i) If the objective is to minimize the average cost per application, determine the number of analysts who should be hired.

ii) What is the minimum average cost of processing an application expected?
Q.3 (b) What will be the Present value of a continuous income strem of Rs. 3500 per annum for four years, if it is discounted continuously at the rate of 0.05 per year.

Q.4 (a) The market demand law of a firm is given by \(4p + q - 16 = 0\), find the output level when the revenue is maximum and find the maximum revenue.

(b) If \( Z = \frac{1}{\sqrt{x^2 + y^2}} \), find partial derivatives of \( Z \) w.r.t. \( x \) and \( y \) respectively.

Q.5. Integrate

(i) \( \int x^4 e^{2x} dx \)

(ii) \( \int \frac{3x^2 - 5x}{(2x^2 - 5x^2 + 15)^2} dx \)

Q.6 (a) Determine the area bounded by the curve \( y = \frac{1}{\sqrt{8-x}} \) and \( x \)-axis and \( x = 4 \).

(b) Given the demand function,

\[ q = \frac{2}{p_1 + p_2 + p_3} = 250,000 - 0.5p_1^2 + p_2^2 - 0.4p_2^3 \]

i) Determine the partial derivatives \( p_{1s}, p_{2s} \) and \( p_{4s} \).

ii) If \( p_1 = 30, p_2 = 10 \) and \( p_3 = 20 \), evaluate the partial derivatives and interpret their meaning.